OCR
RECOGNISING ACHIEVEMENT

**ADVANCED SUBSIDIARY GCE**

# MATHEMATICS (MEI) 4751

Introduction to Advanced Mathematics (C1)

**QUESTION PAPER**

Candidates answer on the printed answer book.

**OCR supplied materials:**
- Printed answer book 4751
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**
None

**Monday 10 January 2011**
**Morning**

**Duration:** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**
These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**
This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

**No calculator can be used for this paper**

 OCR is an exempt Charity

**Section A** (36 marks)

**1** Find the equation of the line which is parallel to $y = 5x - 4$ and which passes through the point (2, 13). Give your answer in the form $y = ax + b$. **[3]**

**2** **(i)** Write down the value of each of the following.

($A$) $4^{-2}$ **[1]**

($B$) $9^0$ **[1]**

**(ii)** Find the value of $\left(\frac{64}{125}\right)^{\frac{4}{3}}$. **[2]**

**3** Simplify $\dfrac{(3xy^4)^3}{6x^5y^2}$. **[3]**

**4** Solve the inequality $5 - 2x < 0$. **[2]**

**5** The volume $V$ of a cone with base radius $r$ and slant height $l$ is given by the formula

$$V = \tfrac{1}{3}\pi r^2\sqrt{l^2 - r^2}.$$

Rearrange this formula to make $l$ the subject. **[4]**

**6** Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of $(2 - 3x)^5$, simplifying each term. **[4]**

**7** **(i)** Express $\dfrac{81}{\sqrt{3}}$ in the form $3^k$. **[2]**

**(ii)** Express $\dfrac{5 + \sqrt{3}}{5 - \sqrt{3}}$ in the form $\dfrac{a + b\sqrt{3}}{c}$, where $a$, $b$ and $c$ are integers. **[3]**

**8** Find the coordinates of the point of intersection of the lines $x + 2y = 5$ and $y = 5x - 1$. **[3]**

**9** Fig. 9 shows a trapezium ABCD, with the lengths in centimetres of three of its sides.

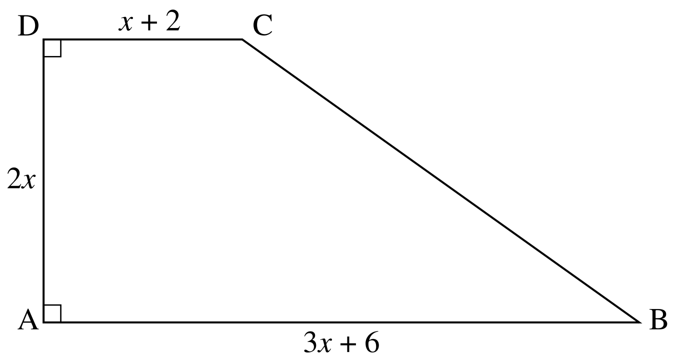


**Fig. 9**

This trapezium has area 140 cm$^2$.

**(i)** Show that $x^2 + 2x - 35 = 0$. **[2]**

**(ii)** Hence find the length of side AB of the trapezium. **[3]**

**10** Select the best statement from

P $\Rightarrow$ Q
P $\Leftarrow$ Q
P $\Leftrightarrow$ Q
none of the above

to describe the relationship between P and Q in each of the following cases.

**(i)** P : WXYZ is a quadrilateral with 4 equal sides
Q : WXYZ is a square

**(ii)** P : $n$ is an odd integer
Q : $(n + 1)^2$ is an odd integer

**(iii)** P : $n$ is greater than 1 and $n$ is a prime number
Q : $\sqrt{n}$ is not an integer **[3]**

## Section B (36 marks)

**11** The points A $(-1, 6)$, B $(1, 0)$ and C $(13, 4)$ are joined by straight lines.

**(i)** Prove that the lines AB and BC are perpendicular. **[3]**

**(ii)** Find the area of triangle ABC. **[3]**

**(iii)** A circle passes through the points A, B and C. Justify the statement that AC is a diameter of this circle. Find the equation of this circle. **[6]**

**(iv)** Find the coordinates of the point on this circle that is furthest from B. **[1]**

**12** **(i)** You are given that $\mathrm{f}(x) = (2x - 5)(x - 1)(x - 4)$.

($A$) Sketch the graph of $y = \mathrm{f}(x)$. **[3]**

($B$) Show that $\mathrm{f}(x) = 2x^3 - 15x^2 + 33x - 20$. **[2]**

**(ii)** You are given that $\mathrm{g}(x) = 2x^3 - 15x^2 + 33x - 40$.

($A$) Show that $\mathrm{g}(5) = 0$. **[1]**

($B$) Express $\mathrm{g}(x)$ as the product of a linear and quadratic factor. **[3]**

($C$) Hence show that the equation $\mathrm{g}(x) = 0$ has only one real root. **[2]**

**(iii)** Describe fully the transformation that maps $y = \mathrm{f}(x)$ onto $y = \mathrm{g}(x)$. **[2]**

**13**

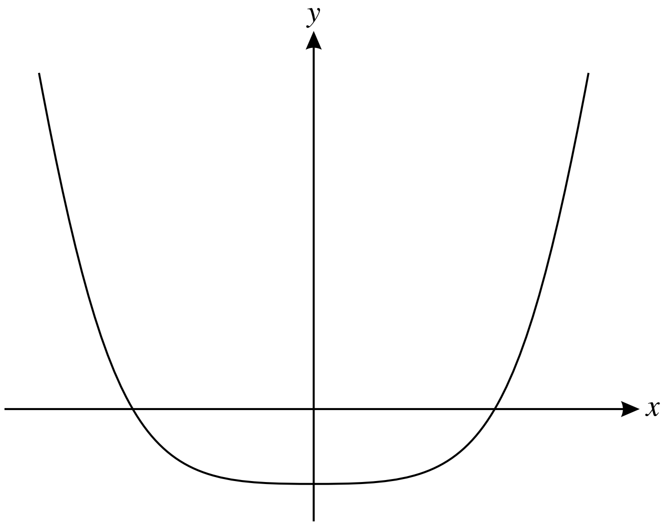


**Fig. 13**

Fig. 13 shows the curve $y = x^4 - 2$.

**(i)** Find the exact coordinates of the points of intersection of this curve with the axes. **[3]**

**(ii)** Find the exact coordinates of the points of intersection of the curve $y = x^4 - 2$ with the curve $y = x^2$. **[5]**

**(iii)** Show that the curves $y = x^4 - 2$ and $y = kx^2$ intersect for all values of $k$. **[2]**

OCR
RECOGNISING ACHIEVEMENT

**ADVANCED SUBSIDIARY GCE**

# MATHEMATICS (MEI) 4751

Introduction to Advanced Mathematics (C1)

**PRINTED ANSWER BOOK**

Candidates answer on this printed answer book.

**OCR supplied materials:**
- Question paper 4751 (inserted)
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**
None

**Monday 10 January 2011**
**Morning**

**Duration:** 1 hour 30 minutes

| Candidate forename | | Candidate surname | |
|---|---|---|---|

| Centre number | | | | | | Candidate number | | | | |
|---|---|---|---|---|---|---|---|---|---|---|

**INSTRUCTIONS TO CANDIDATES**

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- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
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**INFORMATION FOR CANDIDATES**

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- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

**No calculator can be used for this paper**

**Section A** (36 marks)

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|---|---|
| **1** | |
| **2 (i)** (*A*) | |
| **2 (i)** (*B*) | |
| **2 (ii)** | |
| **3** | |

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| **7 (i)** | |
| **7 (ii)** | |
| **8** | |

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|---|---|
| **9 (i)** | |
| **9 (ii)** | |
| **10 (i)** | |
| **10 (ii)** | |
| **10 (iii)** | |

**Section B** (36 marks)

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|---|---|
| **11 (i)** | |
| **11 (ii)** | |

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|---|---|
| **11 (iii)** | |
| **11 (iv)** | |

| | |
|---|---|
| **12(i)**(*A*) | |
| **12(i)**(*B*) | |
| **12(ii)**(*A*) | |

| | |
|---|---|
| **12(ii)**(*B*) | |
| **12(ii)**(*C*) | |
| **12 (iii)** | |

| | |
|---|---|
| **13 (i)** | |
| **13 (ii)** | |

13 (iii)

**PLEASE DO NOT WRITE ON THIS PAGE**

RECOGNISING ACHIEVEMENT

GCE

# Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751:** Introduction to Advanced Mathematics

# Mark Scheme for January 2011

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.



Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
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NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

**Marking instructions for GCE Mathematics (MEI): Pure strand**

1. You are advised to work through the paper yourself first. Ensure you familiarise yourself with the mark scheme before you tackle the practice scripts.

2. You will be required to mark ten practice scripts. This will help you to understand the mark scheme and will not be used to assess the quality of your marking. Mark the scripts yourself first, using the annotations. Turn on the comments box and make sure you understand the comments. You must also look at the definitive marks to check your marking. If you are unsure why the marks for the practice scripts have been awarded in the way they have, please contact your Team Leader.

3. When you are confident with the mark scheme, mark the ten standardisation scripts. Your Team Leader will give you feedback on these scripts and approve you for marking. (If your marking is not of an acceptable standard your Team Leader will give you advice and you will be required to do further work. You will only be approved for marking if your Team Leader is confident that you will be able to mark candidate scripts to an acceptable standard.)

4. Mark strictly to the mark scheme. If in doubt, consult your Team Leader using the messaging system within *scoris*, by email or by telephone. Your Team Leader will be monitoring your marking and giving you feedback throughout the marking period.

   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

5. The following types of marks are available.

   **M**
   A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   **A**
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

   **B**
   Mark for a correct result or statement independent of Method marks.

**E**
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

6. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

7. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

   Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

8. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

9. **Rules for crossed out and/or replaced work**

   If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

   If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

   If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

   If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

10. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

    Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

11. Annotations should be used whenever appropriate during your marking.

    **The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

    For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

12. For answers scoring no marks, you must either award NR (no response) or 0, as follows:

    Award NR (no response) if:
    - Nothing is written at all in the answer space
    - There is a comment which does not in any way relate to the question being asked ("can't do", "don't know", etc.)
    - There is any sort of mark that is not an attempt at the question (a dash, a question mark, etc.)

    The hash key [#] on your keyboard will enter NR.

    Award 0 if:
    - There is an attempt that earns no credit. This could, for example, include the candidate copying all or some of the question, or any working that does not earn any marks, whether crossed out or not.

13. The following abbreviations may be used in this mark scheme.

| | |
|---|---|
| M1 | method mark (M2, etc, is also used) |
| A1 | accuracy mark |
| B1 | independent mark |
| E1 | mark for explaining |
| U1 | mark for correct units |
| G1 | mark for a correct feature on a graph |
| M1 dep* | method mark dependent on a previous mark, indicated by * |
| cao | correct answer only |
| ft | follow through |
| isw | ignore subsequent working |
| oe | or equivalent |
| rot | rounded or truncated |
| sc | special case |
| soi | seen or implied |
| www | without wrong working |

14. Annotating scripts. The following annotations are available:

✓and ✗
**BOD** Benefit of doubt
**FT** Follow through
**ISW** Ignore subsequent working (after correct answer obtained)
**M0, M1** Method mark awarded 0, 1
**A0, A1** Accuracy mark awarded 0, 1
**B0, B1** Independent mark awarded 0,1
**SC** Special case
**^** Omission sign
**MR** Misread
Highlighting is also available to highlight any particular points on a script.

15. The comments box will be used by the Principal Examiner to explain his or her marking of the practice scripts for your information. Please refer to these comments when checking your practice scripts.

    **Please do not type in the comments box yourself.** Any questions or comments you have for your Team Leader should be communicated by the *scoris* messaging system, e-mail or by telephone.

16. Write a brief report on the performance of the candidates. Your Team Leader will tell you when this is required. The Assistant Examiner's Report Form (AERF) can be found on the Cambridge Assessment Support Portal. This should contain notes on particular strengths displayed, as well as common errors or weaknesses. Constructive criticisms of the question paper/mark scheme are also appreciated.

17. Link Additional Objects with work relating to a question to those questions (a chain link appears by the relevant question number) – see scoris assessor Quick Reference Guide page 19-20 for instructions as to how to do this – this guide is on the Cambridge Assessment Support Portal and new users may like to download it with a shortcut on your desktop so you can open it easily! For AOs containing just formulae or rough working not attributed to a question, tick at the top to indicate seen but not linked. When you submit the script, *scoris* asks you to confirm that you have looked at all the additional objects. Please ensure that you have checked all Additional Objects thoroughly.

18. The schedule of dates for the marking of this paper is displayed under 'OCR Subject Specific Details' on the Cambridge Assessment Support Portal. It is vitally important that you meet these requirements. If you experience problems that mean you may not be able to meet the deadline then you must contact your Team Leader without delay.

**SECTION A**

| | | | | |
|---|---|---|---|---|
| 1 | $y = 5x + 3$ | **3** | **M2** for $y - 13 = 5(x - 2)$ oe<br><br>or **M1** for $y = 5x$ [$+ k$] [$k$ = letter or number other than $-4$] and **M1** for $13 =$ their $m \times 2 + k$ | or **M1** for $y - b = 5(x - a)$ with wrong $a$, $b$ or for $y - 13 =$ their $5(x - 2)$ oe<br><br>**M0** for first M if $-1/5$ used as gradient even if 5 seen first; second M still available if earned |
| 2 | (i)(*A*) 1/16 | **1** | isw attempted conversion of 1/16 to decimals | accept 0.0625 |
| 2 | (i)(*B*) 1 | **1** | | set image 'fit to height' so that in marking this question you also check that there is no working on the back page attached to the image |
| 2 | (ii) 256/625 | **2** | **M1** for num or denom correct or for 4/5 or 0.8 | accept 0.4096 |
| 3 | $\frac{9y^{10}}{2x^2}$ oe as final answer | **3** | **1** for each 'term'; 27/6 gets 0 for first term<br><br>if **0**, allow **B1** for $(3xy^4)^3 = 27x^3y^{12}$ | allow eg $4.5x^{-2}y^{10}$ |
| 4 | $x > 5/2$ oe ($-5/-2$ oe not sufft) | **2** | **M1** for $5 < 2x$ or for 5/2 oe obtained with equation or wrong inequality | **M0** for just $-2x < -5$ (not sufft) ; **M1** for $x > -5/-2$ |

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| 5 | $\frac{3V}{\pi r^2} = \sqrt{l^2 - r^2}$<br><br>$\left(\frac{3V}{\pi r^2}\right)^2 = l^2 - r^2$<br><br>$l^2 = \left(\frac{3V}{\pi r^2}\right)^2 + r^2$<br><br>$[l =]\sqrt{\left(\frac{3V}{\pi r^2}\right)^2 + r^2}$ | **M1**<br><br>**M1**<br><br>**M1**<br><br>**M1** | for correctly getting non- '$l^2 - r^2$' terms on other side[**M0** for 'triple decker' fraction]<br><br>oe or ft; for squaring correctly<br><br>oe or ft; for getting $l$ term as subject<br><br>oe. or ft; mark final answer; for finding square root ( and dealing correctly with coefficient of $l$ term if needed at this stage); condone ±√etc | may be done in several steps, if so, condone omission of brackets in eg $9V^2 = \pi^2 r^4\ l^2 - r^2$ if they recover – if not, do not give 1st **M1** [but can earn the 2nd **M1**]<br><br>for combined steps, allow credit for correct process where possible;<br><br>eg $\pi^2 r^4 l^2$ as the term on one side<br><br>For **M4**, the final expression must be totally correct, [condoning omission of $l$ and insertion of ±]<br>eg **M4** for $\frac{\sqrt{9V^2 + \pi^2 r^6}}{\pi r^2}$ |
| 6 | $32 - 240x + 720x^2$ isw | **4** | **B3** for all correct except for sign error(s)<br>**B2** for 2 terms correct numerically, ignoring any sign error or for 32, –240 and 720 found<br>or B2 for all correct, including signs, but unsimplified<br>**B1** for binomial coeffts 1, 5, 10 used or 1 5 10 10 5 1 seen<br><br>**SC3** for $- 240x + 720x^2 - 1080x^3$ isw or for $-243x^5 + 810x^4 - 1080x^3$<br>or **SC2** for these terms with sign error(s) | accept terms listed separately; condone $-240x^1$<br><br>expressions left in ${}^nC_r$ form or with factorials not sufft |

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| 7 | (i) $3^{7/2}$ oe or $k = 7/2$ oe | 2 | **M1** for $\frac{3^4}{\sqrt{3}}$ or $\frac{81}{3^{1/2}}$ or $81\times3^{-1/2}$ or $3^3\sqrt{3}$ or $27 \times 3^{1/2}$ or better or for $81 = 3^4$ or $\sqrt{3} = 3^{1/2}$ or $\frac{1}{\sqrt{3}} = 3^{-1/2}$ or (following correct rationalisation of denominator) for $27 = 3^3$<br><br>isw conversion of 7/2 oe | **M0** for just $81 = 3 \times 3 \times 3 \times 3$ oe – indices needed<br><br>allow an M mark for partially correct work still seen in fraction form eg $\frac{3^4}{3^{-1/2}}$ gets mark for $81 = 3^4$ |
| 7 | (ii) $\frac{14+5\sqrt{3}}{11}$ or $\frac{28+10\sqrt{3}}{22}$ www isw | 3 | **M1** for multiplying num and denom by $5 + \sqrt{3}$<br>and **M1** for num or denom correct in final answer (M0 if wrongly obtained) | 2nd **M1** is not dependent on 1st **M1** |
| 8 | (7/11, 24/11) oe www | 3 | **B2** for one coord correct; condone not expressed as coords, isw<br><br>or **M1** for subst or elimination; eg $x + 2(5x - 1) = 5$ oe; condone one error<br><br>**SC2** for mixed fractions and decimals eg (3.5/5.5, 12/5.5) | |
| 9 | (i) $\frac{1}{2} \times 2x \times (x + 2 + 3x + 6)$ oe<br><br>$x(4x + 8) = 140$ oe and given ans $x^2 + 2x - 35 = 0$ obtained correctly with at least one further interim step | M1<br><br>A1 | correct statement of area of trap; may be rectangle ± triangle, or two triangles | eg $2x(x + 2) + \frac{1}{2} \times 2x \times (2x + 4)$<br><br>condone missing brackets for **M1;** condone also for **A1** if expansion is treated as if they were there |

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| | (ii) [AB =] 21 www | **3** | or **B2** for $x$ = [−7 or] 5 cao www or for AB = 21 or −15<br><br>or **M1** for $(x + 7)(x - 5)$ [= 0]or formula or completing square used eg $(x + 1)^2 - 36$ [= 0]; condone one error eg factors with sign wrong or which give two terms correct when expanded<br><br>or **M1** for showing f(5) = 0 without stating $x = 5$ | may be done in (i) if not here – allow the marks if seen in either part of the image – some candidates are omitting the request in (i) and going straight to solving the equation (in which case give 0 [not NR] for (i), but annotate when the image appears again in (ii))<br><br>5 on its own or AB = 5 with no working scores 0; we need to see $x = 5$ |
| 10 | (i) P ⇐ Q<br><br>(ii) none [of the above]<br><br>(iii) P ⇒ Q | **1**<br><br>**1**<br><br>**1** | or ⇐ or ‘Q ⇒ P’<br><br><br><br>or ⇒ | Condone single arrows |

Section A Total: 36

**SECTION B**

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| 11 | (i) grad AB = $\frac{0-6}{1-(-1)}$ oe [= −3] isw<br><br>grad BC = $\frac{0-4}{1-13}$ oe [= 1/3] isw<br><br>product of grads = −1 [so lines perp] stated or shown numerically | **M1**<br><br>**M1**<br><br>**M1** | for full marks, it should be clear that grads are independently obtained<br><br>or 'one grad is neg. reciprocal of other'<br><br>**or**<br>**M1** for length of one side (or square of it)<br>**M1** for length of other two sides (or their squares) found independently<br>**M1** for showing or stating that Pythag holds [so triangle rt angled] | eg grads of −3 and 1/3 without earlier working earn **M1M0**<br><br>for **M3**, must be fully correct, with gradients evaluated at least to −6/2 and −4/−12 stage<br><br>$AB^2 = 6^2 + 2^2 = 40$, $BC^2 = 4^2 + 12^2 = 160$, $AC^2 = 14^2 + 2^2 = 200$ |
| 11 | (ii) AB = √40 or BC = √160<br><br>½ × √40 × √160 oe or ft their AB, BC<br><br>40 | **M1**<br><br>**M1**<br><br>**A1** | **or M1** for one of area under AC (=70), under AB (=6) under BC (=24) (accept unsimplified) and **M1** for their trap. − two triangles | allow **M1** for $\sqrt{(1-(-1))^2+(6-0)^2}$ or for $\sqrt{(13-1)^2+(4-0)^2}$<br><br>**or** for rectangle – 3 triangles method,<br>[ $6\times14-\frac{1}{2}(2)(6)-\frac{1}{2}(4)(12)-\frac{1}{2}(2)(14)$<br>=84 – 6 – 24 – 14]<br>**M1** for two of the 4 areas correct and **M1** for the subtraction |

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| 11 | (iii) angle subtended by diameter = 90° soi<br><br>mid point M of AC = (6, 5)<br><br>rad of circle = $\frac{1}{2}\sqrt{14^2+2^2}\,[=]\,\frac{1}{2}\sqrt{200}$ oe or equiv using $r^2$<br><br>$(x-a)^2+(y-b)^2=r^2$ seen or $(x - \text{their } 6)^2 + (y - \text{their } 5)^2 = k$ used, with $k > 0$<br><br>$(x-6)^2+(y-5)^2=50$ cao | B1<br><br>B2<br><br>M1<br><br>M1<br><br>A1 | or angle at centre = twice angle at circumf = 2 × 90 = 180 soi<br>or showing BM = AM or CM, where M is midpt of AC; or showing that BM = ½ AC<br><br>allow if seen in circle equation ; **M1** for correct working seen for **both** coords<br><br>accept unsimplified; or eg $r^2 = 7^2 + 1^2$ or $5^2 + 5^2$; may be implied by correct equation for circle or by correct method for AM, BM or CM ft their M<br><br>or $x^2+y^2-12x-10y+11=0$ | condone 'AB and BC are perpendicular' or 'ABC is right angled triangle' provided no spurious extra reasoning<br><br>allow **M1** bod intent for AC = $\sqrt{200}$ followed by $r = \sqrt{100}$<br><br>must be simplified (no surds) |
| 11 | (iv) (11, 10) cao | 1 | | |
| 12 | (i)(A) sketch of cubic correct way up and with two tps, crossing $x$-axis in 3 distinct points<br><br>crossing $x$ axis at 1, 2.5 and 4<br><br>crossing $y$ axis at −20 | B1<br><br>B1<br><br>B1 | **0** if stops at $x$-axis; condone not crossing $y$-axis<br><br>intersections labelled on graph or shown nearby in this part; mark intent for intersections with both axes (eg condone graphs stopping at axes)<br><br>or $x = 0$, $y = -20$ seen in this part if consistent with graph drawn | No section to be ruled; no curving back; condone slight 'flicking out' at ends; condone some doubling (eg erased curves may continue to show)<br><br>allow 2.5 indicated by graph crossing halfway between their marked 2 and 3 on scale; allow if no graph but 0 if graph inconsistent with values<br><br>allow if no graph, but eg **B0** for graph with intn on +ve $y$-axis or nowhere near their indicated −20 |

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| 12 | (i)(B) correct expansion of two brackets<br><br>correct interim step(s) multiplying out linear and quadratic factors before given answer<br><br>**or**<br>showing that 1, 2.5 and 4 all satisfy $f(x) = 0$ for cubic in $2x^3$ ... form<br><br>comparing coeffts of eg $x^3$ in the two forms | **M1**<br><br>**M1**<br><br>**or**<br>**M1**<br><br>**M1** | or **M2** for all 3 brackets multiplied at once, showing all 8 terms (**M1** if error in one term): $2x^3 - 8x^2 - 2x^2 - 5x^2 + 8x + 5x + 20x - 20$<br><br>**or**<br>**M1** for dividing $2x^3$ ... form by one of the linear factors and **M1** for factorising the resultant quadratic factor | eg **M1** for $(2x - 5)(x^2 - 5x + 4)$<br><br>condone missing brackets if intent clear /used correctly |
| 12 | (ii)(A) 250 − 375 + 165 − 40 isw | **B1** | **or**<br>showing that $x - 5$ is a factor by eg division and then stating that $x = 5$ is root or that $g(5) = 0$ | '2 × 125 + 15 × 25 + 33 × 5 – 40' is not sufft<br><br>**or**<br>[g(5) =] f(5) – 20 = 5 × 4 × 1 – 20 [= 0] |
| 12 | (ii) (B) $(x - 5)$ seen or used as linear factor<br><br>division by $(x - 5)$ as far as $2x^3 - 10x^2$ seen in working<br><br>$2x^2 - 5x + 8$ obtained isw | **M1**<br><br>**M1**<br><br>**A1** | may be in attempt at division<br><br>or inspection/equating coefficients with two terms correct eg $(2x^2 \ldots\ldots + 8)$<br><br>eg may be seen in grid;<br><br>condone $g(x)$ not expressed as product | allow if seen in (ii)(A)<br><br>for division: condone signs of $2x^3 - 10x^2$ changed for subtraction, or subtraction sign in front of first term |

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| 12 | (ii)(C) $b^2 - 4ac$ used on their quadratic factor<br><br>$(-5)^2 - 4 \times 2 \times 8$ oe and negative [or $-39$] so no [real] root [may say only one [real] root, thinking of $x = 5$] | **M1**<br><br>**A1** | may be in formula<br><br>[or allow **2** marks for complete correct attempt at completing square and conclusion, or using calculus to show min value is above $x$-axis and comment re curve all above $x$-axis] | no ft for A mark from wrong quadratic factor<br><br>condone error in working out –39 if correct unsimplified expression seen and neg result obtained<br><br>$-5^2 - 4 \times 2 \times 8$ evaluated correctly with comment is eligible for **A1**, otherwise bod for the **M1** only |
| 12 | (iii) translation<br><br>$\begin{pmatrix} 0 \\ -20 \end{pmatrix}$ | **B1**<br><br>**B1** | NB 'Moves' not sufficient for this first mark<br><br>or 20 down; | <br><br>**B0** for second mark if choice of one wrong, one right description |
| 13 | (i) (0, −2) or 'crosses $y$-axis at −2' oe isw<br><br>$(\pm 2^{\frac{1}{4}}, 0)$ oe isw | **B1**<br><br>**B2** | <br><br>or [when $y = 0$],<br>$[x =] \pm 2^{\frac{1}{4}}$ or $\pm\sqrt{\sqrt{2}}$ or $\pm\sqrt[4]{2}$ isw<br><br>**B1** for one root correct | condone $y = -2$ |

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| 13 | (ii) [$y =$ ] $x^2 = x^4 - 2$ oe and rearrangement to $x^4 - x^2 - 2$ [= 0] or $y^2 - y - 2$ [=0] | **M1** | | |
| | $(x^2 - 2)(x^2 + 1) = 0$ oe in $y$ | **M1** | or formula or completing square; condone one error; condone replacement of $x^2$ by another letter or by $x$ for 2nd M1 (but not the 3rd M1) | if completing square, and haven't arranged to zero, can earn first **M1** as well for an attempt such as $(x^2 - 0.5)^2 = 2.25$ |
| | $x^2 = 2$ [or −1] or $y = 2$ or −1 or ft or $x = \sqrt{2}$ or $x = -\sqrt{2}$ or ft | **M1** | dep on 2nd **M1;** allow inclusion of correct complex roots; **M0** if any incorrect roots are included for $x^2$ or $x$ | NB for second and third M: **M0** for $x^2 - 2 = 0$ or $x^2 = 2$ oe straight from quartic eqn – some candidates probably thinking $x^4 - x^2$ simplifies to $x^2$; last two marks for roots are available as B marks |
| | $(\sqrt{2}, 2)$ and $(-\sqrt{2}, 2)$; with no other intersections given | **B2** | or **B1** for one of these two intersections (even if extra intersections given) or for $x = \pm\sqrt{2}$ (and no other roots) or for $y = 2$ (and no other roots), marking to candidates' advantage | some candidates having several attempts at solving this equation – mark the best in this particular case |

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| 13 | (iii) from $x^4 - kx^2 - 2$ [= 0]:<br><br>$k^2 + 8 > 0$ oe<br><br>$k + \sqrt{k^2+8} \geq 0$ for all $k$<br><br>[so there is a positive root for $x^2$ and hence real root for $x$ and so intersection] | <br><br>**B1**<br><br>**B1** | Allow $x^2$ replaced by other letters or $x$ or from $y^2 - k^2y - 2k^2$ [= 0]<br><br>$k^4 + 8k^2 > 0$ oe<br><br>$k^2 + \sqrt{k^4 + 8k^2} > 0$ oe for all $k$<br><br>[so there is a positive root for $y$ and hence real root for $x$ and so intersection]<br><br>if **B0B0**, allow **SC1** for $\frac{k \pm \sqrt{k^2+8}}{2}$ or $\frac{k^2 \pm \sqrt{k^4+8k^2}}{2}$ obtained [need not be simplified] | [alt methods: may use completing square to show similarly, or comment that at $x = 0$ the quadratic is above the quartic and that as $x \to \infty$, $x^4 - 2 > kx^2$ for all $k$]<br>condone lack of brackets in $(-k)^2$ |

Section B Total: 36

# 4751 Introduction to Advanced Mathematics

### General Comments

Overall the candidates coped well and it seemed that there were few candidates who clearly were not ready to take the paper. Elegant work was seen from some of the strong candidates. There were some challenging questions to stretch the brightest of candidates, and the grade boundaries reflect this. Although the last part of the last question was omitted by some candidates, examiners felt that this was due to lack of knowledge as to how to proceed, rather than any time problem.

It was evident that surds caused problems to many candidates in questions 11 and 7(i), yet the routine work in question 7(ii) was done well, suggesting that candidates are being well trained to cope with rationalising fractions with surds in the denominator.

The lack of maturity in algebraic manipulation was also evident in many candidates' work, with the last mark in question 5 often lost by 'rooting' individual terms, whilst 13(ii) was often spoilt by 'simplifying' $x^4 - x^2 - 2$ to $x^2 - 2$. The lack of correct use of brackets in their work led to errors by some candidates, for instance in questions 5 and 6, whilst incorrect arithmetic caused problems in question 8, for example, where some candidates could not obtain the correct value for $y$ when substituting a fractional value for $x$.

### Comments on Individual Questions

### Section A

**1)** Finding the equation of the parallel line proved an easy starter as anticipated. Most obtained 3 marks without any difficulty, with very few making arithmetic errors here. Only a small minority used the perpendicular gradient, but even they picked up a mark for substituting (2, 13) correctly into their equation.

**2)** The first part was answered well, although a few answers of $\frac{1}{4}$ or $\frac{1}{8}$ or 16 were seen. In the second part, nearly all candidates knew that $a^0 = 1$.

**3)** Only about 30% of candidates scored all 3 marks here, with many failing to simplify $\frac{27}{6}$ in spite of the request. Common errors with the powers were $(y^4)^3 = y^7$ and $3^3 = 9$.

**4)** This was mostly well done. Of those who failed to get both marks, many scored M1 for getting 2.5, but with the wrong inequality. Candidates who began with $5 < 2x$ were generally more successful than those who started with $-2x < -5$.

**5)** There were many poor attempts at changing the subject of this formula, although plenty of good solutions were also seen. Most of the confusion arose from an inability to deal correctly with the square roots at the beginning and at the end. Many made errors in the initial squaring by failing to use brackets, whilst the error in the last step was due to thinking that $\sqrt{a^2 + b^2} = a + b$. Follow-through marks for correct work following earlier errors helped candidates to gain partial credit.

**6)** A minority of candidates did this question well; gaining only 2 out of 4 marks was common. A common mistake was not to bracket $-3x$ and then to square it incorrectly. It was particularly evident in this question that many candidates would benefit from setting out their work more clearly. There were fewer attempts this time to obtain the result by multiplying out instead of by using the binomial theorem, no doubt partly because that would have been so lengthy a process. The special case marks enabled credit to be given to candidates who did not answer the question precisely.

**7)** In part (i) it was evident that only a minority were comfortable with negative and fractional indices and knew that $81 = 3^4$, though some worked that out. Many never converted to index form at all.
Part (ii) was done well and full marks were common. Candidates knew how to rationalise the denominator and most successfully multiplied out the surds.

**8)** Nearly all candidates knew how to find the intersection of the lines Over half of the candidates reached $11x = 7$, and the majority of these $x = \frac{7}{11}$, but a significant proportion then were unable to obtain $y$ correctly. About a third of the candidates gained all 3 marks.

**9)** Failure to read the question was often seen here as many candidates immediately set about solving the quadratic and ignored what was required in part (i). Those that did attempt to produce an expression for the area were usually successful if they knew the formula for the area of a trapezium and less frequently so if they used a method involving splitting the area or subtraction. One type of mistake was to try to contort an incorrect expression rather than go back to the start. Most did obtain the correct answer for the length of AB as required for part (ii). The equation was usually solved by factorisation, but $x = 5$ was not difficult to find by inspection. Most realised that lengths had to be positive and so discarded the $x = -7$ found by factorisation when calculating AB.

**10)** This question was easy to mark and more difficult to do, needing more care and probably time than many gave it. Plenty of good candidates only obtained 1 or 2 marks here, and the full 3 marks was unusual.

## Section B

**11** **(i)** Most candidates made a good attempt at this part, coping with the negative signs in the arithmetic. A majority went on to show that the product of their gradients was –1 and therefore concluded that the lines were perpendicular. Some simply quoted the result, e.g that one gradient was the negative reciprocal of the other or simply that $m_1 \times m_2 = -1$, and these were accepted. A small number of candidates calculated their gradients as change in $x$ / change in $y$ and thought that they had achieved the correct result by a legitimate method. The alternative method of using Pythagoras was very rarely seen.

**(ii)** This part was also attempted well by most candidates, but a large number were unable to cope with the final calculation involving surds, making errors such as $2\sqrt{10} \times 2\sqrt{10} = 4\sqrt{10}$ ,and thus earning just the two method marks. The most common error of method for the area of the triangle was to use the product of the lengths of lines AB and AC; a small number of candidates tried treating the triangle as if it were isosceles, finding a midpoint and attempting to find a height.

**(iii)** Not many candidates were able to give clear and concise reasons for the justification for AC being a diameter. However some referred to the perpendicular lines and others based an argument around the fact that triangle ABC was right-angled which enabled them to gain credit. Some other candidates found the distance of point B from the centre of the circle and then compared this with the radius or the diameter of the circle. Attempts at the equation of the circle were generally better and most candidates managed to get some marks here. Finding the centre of the circle was usually done correctly and many candidates demonstrated that they knew what form the equation of the circle should take, either by quoting a general form or by using the coordinates of their centre e.g. $(x-6)^2 + (y-5)^2 = .....$
However, determining the radius of the circle and hence completing the equation was done less satisfactorily. Once again it was the arithmetic involving surds which let them down e.g. $\frac{\sqrt{200}}{2} = \sqrt{100}$ . As a consequence only a few candidates obtained full marks for this part, but marks of 4 or 5 out of 6 were quite common.

**(iv)** This part was not done well and the majority of candidates had little idea of a suitable strategy. However, some of the stronger candidates coped well with this part.

**12** **(i)** Candidates usually managed to sketch the graph of the cubic in the correct sense, although the quality of the curves drawn was sometimes poor, with 'flicking-out' at the ends. The *x*-intercepts were commonly well found – although some confused themselves with the non-integer intercept, indicating its position incorrectly on the graph. The *x*-intercepts were also sometimes given as 1, 4 and 5 as the 5 had not been divided by 2. Many omitted to work out and mark the position of the *y* – intercept.

**(ii)** Again, this was generally well answered. Most candidates chose to multiply out two of the linear factors to obtain a quadratic, usually correctly. Although it was very common for students to omit relevant brackets, they did usually "recover" to obtain a correct, unsimplified form of the answer. Surprisingly many did not collect *x* terms after multiplying their first pair of brackets, giving 8 terms to be tidied up instead of 6. Candidates who multiplied out all three brackets at once, rather than getting a quadratic factor first, were generally less successful.

The small number of candidates who opted for the method of showing that 1, 2.5 and 4 were roots of the given cubic were rarely successful. Most managed to show that 1 and 4 were roots, but the arithmetic of showing that 2.5 was a root was beyond these (usually weaker) candidates.

The (again) small number of candidates who chose to divide the cubic by one of the linear factors were often not convincing enough in factorising the resultant quadratic to score both marks.

**(iii)** Part *A* was generally very well answered by the expected method of substituting 5. A very small number of (usually good) candidates opted for using $f(5) - 20$. A very small number of candidates opted to divide at this stage, but usually did not complete the argument that $x - 5$ being a factor meant that $x = 5$ was a root.

Algebraic division or comparison of coefficients method were both popular choices of method offered by candidates in part *B*. Very often candidates were successful with their efforts, with only a few candidates making occasional sign or arithmetic errors. It was clear that a small number of candidates did not know where to begin, however, as a few did not attempt this part. Attempts to divide by $(2x - 5)$ or $(x - 1)$ were seen occasionally.

It was pleasing in part *C* to see that most candidates seemed to appreciate that the nature of roots of a quadratic was determined by the discriminant (even though often this was still embedded in the quadratic formula). Most candidates were able to show that the discriminant was negative and give a correct conclusion, but poor arithmetic meant that 25 – 64 was often not –39. Those who did not manage to achieve an answer for part *B* usually attempted to apply the discriminant to the cubic equation, if they made any attempt here.

**(iv)** The word 'translation' was frequently missing from candidates' descriptions of the transformation, with the resultant loss of a mark. '20 down' was common (and accepted), and the majority of those who opted to give a vector were correct – although among the incorrect offerings $\begin{pmatrix} 0 \\ 20 \end{pmatrix}$ or $\begin{pmatrix} -20 \\ 0 \end{pmatrix}$ were common.

**13** As expected, question 13 as a whole was the question on the paper that candidates found the most challenging, with few gaining marks on part (iii).

**(i)** A good number of candidates did score the 3 marks in this part, but many missed the *y*-intercept or, more often, the negative *x*-intercept.

**(ii)** It was very common to see candidates 'simplifying' $x^4 - x^2$ to $x^2$. These candidates scored no further method marks but were able to pick up a maximum of 2 marks if they went on to find the intersections correctly. Those attempting to solve the correct quadratic often gained only part marks since they only took the positive root of 2 or tried to use $\sqrt{-1}$ for the coordinates.

**(iii)** A few of the best candidates got as far as showing that $k^2 + 8$ was always positive, but in most cases they thought that this was all that was required, and did not go on to show that at least one of the roots for $x^2$ must be positive in order for there to be a real root for *x*. A few candidates tried a graphical approach but their explanations were rarely rigorous enough to gain credit.

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